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PREDICTION OF PRESSURE DROP DURING FORCED CIRCULATION BOILING OF WATER

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Abstract—A simplified scheme is given for the calculation of pressure drop during the circulation of a two-phase mixture of boiling water and steam. The method follows that proposed by Martinelli and Nelson which has been extended to include the gravitational term in vertical evaporating tubes. Curves are given from which frictional, acceleration and gravitational losses can be estimated provided the outlet quality has been calculated from a heat balance. These curves are based directly on the experimental result of the boiler circulation research sponsored at the University of Cambridge by the Water-Tube Boilermakers' Association.

An Appendix gives data for calculating the pressure drop during the two-phase flow of water and steam along an unheated tube.

NOMENCLATURE

The symbols used in the various equations are defined as they are introduced into the text. For ready reference, however, they are gathered together in the following list.

Subscripts f, g, and fg indicate a state, or change of state, of the fluid in the pipe. Thus f indicates the water phase, g the steam phase and fg the change from water to steam.

Subscripts H and S refer to methods of calculation. H indicates that the quantity was deduced by homogeneous flow theory while S indicates that slip from theory was used.

The units used are the foot, pound mass and second. Pressures are quoted in psia, and pressure differences in pounds force per ft^2.

- A, cross-sectional area of test-section [ft^2];
- A_g, part of cross-section occupied by steam phase [ft^2];
- D, bore of test pipe [ft];
- D', bore of test pipe [in];
- f, Fanning friction factor for single phase flow;
- G, mass velocity [lb/ft^2 s];
- G', mass velocity [lb/ft^2 h];
- g, local acceleration due to gravity [ft/s^2];
- g_c, dimensionless conversion factor lb force to pdl $= 32{\cdot}2$;
- h_f, enthalpy of water at saturation temperature [Btu/lb];
- h_i, enthalpy of water at inlet temperature [Btu/lb];
- h_{fg}, change of enthalpy with change of state [Btu/lb];
- K, dimensionless flow parameter (Bankoff);
- l, length of test pipe [ft];
- l_i, preheating length of test pipe [ft];
- m, total mass flow rate [lb/s];
- m_f, mass flow rate of water phase [lb/s];
- m_g, mass flow rate of steam phase [lb/s];
- p, pressure of fluid [psia];
- Q, heat addition to fluid along total length of tube [Btu/lb];
- R, expansion ratio between inlet and outlet;
- r_1, multiplier for homogeneous flow acceleration pressure drop [ft^3/lb];
- r_2, multiplier for two-phase flow acceleration pressure drop;
- r_3, multiplier for two-phase flow frictional drop;
- r_4, multiplier for two-phase flow gravitational drop;
- T_i, temperature of water at inlet to tube [°F];
- V, mean bulk velocity of fluid [ft/s];

V_f, V_g, mean velocity of water and steam phases [ft/s];
v, mean bulk specific volume [ft^3/lb];
v_i, specific volume of water at inlet of pipe [ft^3/lb];
v_f, v_g, specific volume of saturated water and steam [ft^3/lb];
v_{fg}, change of specific volume between phases [ft^3/lb];
$\bar{x}_a$, area dryness or void fraction of mixture (fraction of cross-section occupied by vapour phase);
x_m, "mass" dryness fraction of mixture;
x_v, "volumetric" flow ratio of mixture;
Δp, pressure drop along tube [lb/ft^2];
Δp_{acl}, acceleration pressure drop [lb/ft^2];
Δp_f, friction pressure drop for "all water" flow $\dfrac{4fv_f}{2g_c D}(l - l_i)\, G^2$ [lb/ft^2];
Δp_{TPF}, friction pressure drop in two-phase flow [lb/ft^2];
Δp_{grav}, gravitational pressure drop up vertical tube [lb/ft^2];
α, specific volume ratio (v_g/v_f);
γ, dimensionless slip factor (see text);
ρ, density of mixture [lb/ft^3].

IN A NUMBER of engineering flow systems it is important to be able to predict the pressure drop occurring during the forced circulation boiling of water. In these calculations difficulty arises from the fact that the two phases, water and steam, cannot be considered as a uniform homogeneous mixture due to the slip between the phases which takes place under certain conditions. The estimation of the friction factor also presents some difficulty although, as will be shown, the effect of this is of lesser importance.

Martinelli and Nelson [1] proposed a tentative solution based on a number of hypotheses requiring experimental verification and expressed the hope that their paper would stimulate discussion and further investigation of the problem. Zmola and Bailey [2] in their paper "Power removal from boiling nuclear reactors" quoted values copied directly from Martinelli and Nelson, these being the best available at the time. Haywood [3] in his contribution to the General Discussion on Heat Transfer (1951) also drew attention to the need for a more accurate solution to the problem.

Resulting from this the Water-Tube Boilermakers' Association sponsored a full scale project which was carried through the years 1951–59 in the University Engineering Laboratory at Cambridge (England). Some practical aspects of this research have been reported [4] and the experimental findings have been published in detail by Haywood *et al.* [5]. It is felt, however, that many engineers would welcome a simplified method provided (a) it was applicable over a wide range of operating pressures (b) was not restricted to a limited range of steam quality (c) was soluble by ordinary slide rule calculation. The analysis now presented presented covers the pressure range 15–3000 psia and outlet steam qualities 3–100 per cent. The method follows the lines suggested by Martinelli and Nelson for a horizontal tube but has been extended to include the much more important case of a vertical evaporating tube, a condition not covered by that analysis. For this purpose the wider range of the earlier Cambridge data [6] proved useful. In that part of the investigation void fraction measurements were made using a gamma-source and ionization chamber. Figure 1, which is comparable to Fig. 51 of reference 5, shows some of these

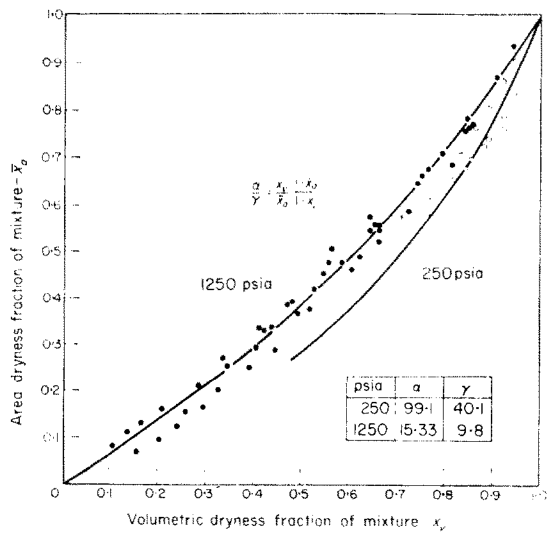

FIG. 1. Relation between volumetric and area dryness fraction, two-phase flow in 1 in bore horizontal pipe.

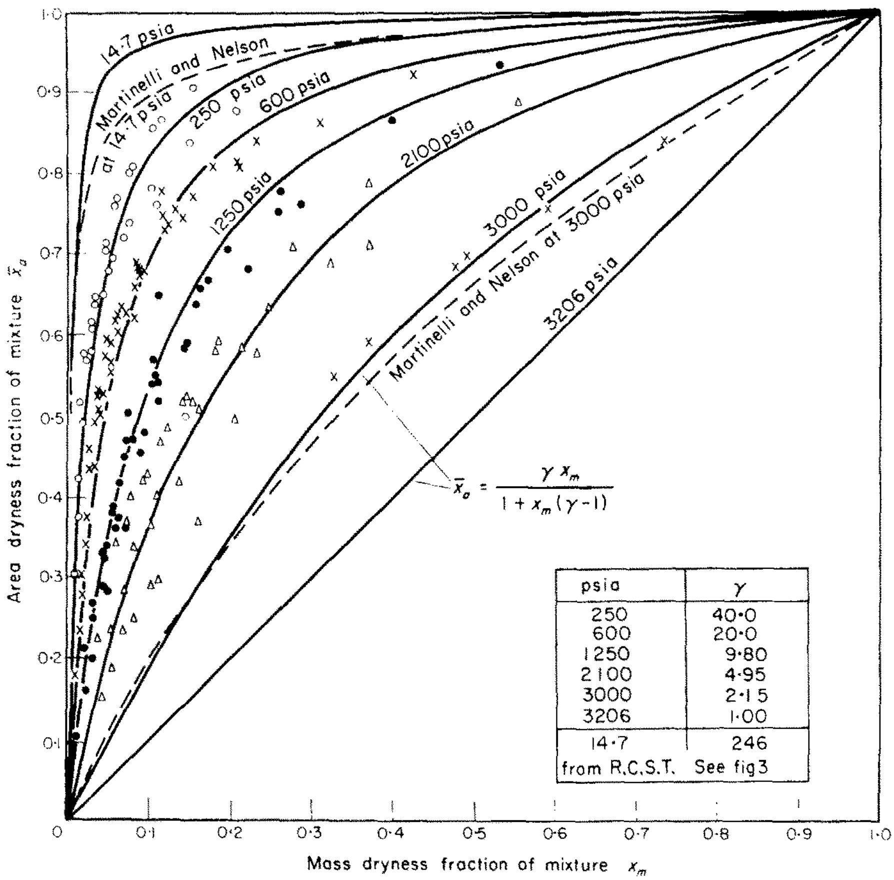

FIG. 2. Relation between area and mass dryness fraction, two-phase flow in 1 in bore horizontal pipe.

observations. The analysis now presented is an attempt to facilitate extrapolation to regions of higher steam quality and Fig. 2 shows similar data plotted against mass dryness fraction. This conceals the experimental scatter at low steam qualities shown in Fig. 1 but opens out the scale in the range towards which this paper is directed, i.e. mass dryness fractions greater than 3 per cent. Further, it is appreciated that the empirical relationship equation (5) between "mass dryness" and "void" fraction is a compromise and implies a constant slip velocity ratio at any given operating pressure. However, the method leads to a relatively easy method of calculating boiling two-phase flow pressure drop and, except at very low steam qualities, appears to predict reasonable values (Figs. 4, 5 and 12).

REGIMES OF FLOW

When a fluid is flowing along a heated tube, the heat transfer and flow behaviour varies with the heat flux and condition of the fluid. In particular there appear to be three distinct regimes of flow:

(a) When the fluid temperature and the heat flux are low and convective heat transfer takes place across the boundary layer without change of phase.

(b) When either the fluid temperature or the heat flux is high enough for the wall temperature to rise a few degrees above boiling point, nucleate boiling takes place, the bubbles being swept off the surface by the flow of fluid.

(c) At high heat fluxes the bubbles near the surface become closely packed until, at a certain critical value, a continuous vapour film is formed on the surface.

The work described here is concerned with the second of these three regimes. Care was taken to ensure that the water entered the test-section exactly at saturation temperature and heat transfer took place with net generation of steam. Since the fluid bulk temperature was already at saturation these bubbles did not condense again but caused an increase in volume and velocity of the mixture. This resulted in increased pressure loss. In addition, there was some evidence of increase in friction pressure loss thought to be due to the effective roughness of the tube surface being altered by the bubble formation. This effect, however, was of minor importance in comparison with the acceleration and gravitational pressure losses.

It is postulated that, when the pressure drop across the tube is small compared to the absolute pressure, the pressure drop resulting from the flow of a boiling mixture along a horizontal tube is made up of two parts:

(a) the pressure drop resulting from the increase in momentum of the mixture as it flows through the tube and vaporizes.
(b) the pressure drop due to the frictional forces acting during two-phase flow.

In the case of a vertical tube a third part (c) becomes important, i.e. the pressure drop due to the increase in gravitational potential energy as the boiling mixture rises in the tube.

In experimental work it is not possible to measure directly the individual parts of the overall pressure drop. Thus, a theory has been built up, whereby that part of the total pressure drop due to acceleration only, can be assessed. This theory was then extended to give the gravitational pressure drop in a vertical evaporating tube. The sum of these two parts was deducted from the total observed pressure drop and the remainder assumed to have been caused by friction.

Within the pressure range from 250 psia upward the difference between the deduced two-phase frictional pressure drop and that predicted by homogeneous flow theory is small but the components of the total pressure drop due to inertia and especially to buoyancy effects in a vertical tube are affected by the steam slip relative to water. This effect is noticeable, even at high pressures, in the difference between the void fraction and the volumetric flow ratio.

METHODS OF ASSESSING QUALITY

Since it is desirable to know the quality of the two-phase mixture passing the outlet, it is important to appreciate the various ways in which this can be expressed.

The term "mass" dryness fraction is in general use and is defined by

$$x_m = \frac{m_g}{m_g + m_f} \tag{1}$$

in which m_g and m_f are respectively the mass of steam and water at the outlet. Usually the value of x_m is calculated from a heat balance.

An alternative method is to compare by volume instead of by mass. In this case the "volumetric" flow ratio is defined by

$$x_v = \frac{x_m v_g}{x_m v_g + (1 - x_m)\, v_f}. \tag{2}$$

Ideally, these two definitions are applicable only to a homogeneous mixture in which both steam and water phases flow at the same rate. The effect of "slip" however is to cause the steam phase to travel a little faster than the homogeneous mean and to occupy a smaller part of the cross-section of the tube. Conversely, the water phase travels a little slower than the homogeneous mean and occupies a larger part of the cross-section than would be expected. This condition was investigated by a gamma-ray beam which was arranged to scan the tube cross-section at the outlet as described by several workers [4, 5, 6]. From these observations a third definition called the "void" fraction has been deduced and is defined by

$$\bar{x}_a = \frac{A_g}{A} \tag{3}$$

in which A is the cross-section of the tube and A_g that part of the cross-section occupied by the steam phase.

Equation (2) may be written

$$x_v = \frac{a \,.\, x_m}{1 + x_m\,(a - 1)} \tag{4}$$

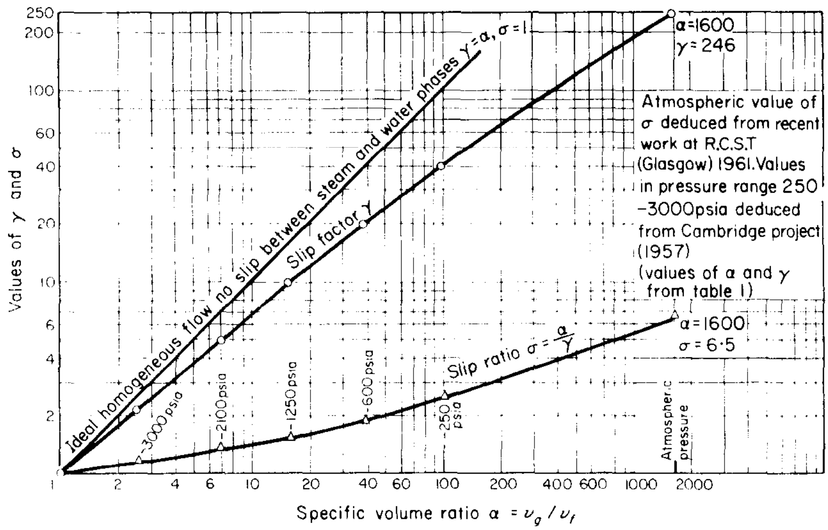

FIG. 3. Two-phase flow of water and steam. Experimentally determined values of γ and σ.

in which the specific volume ratio $\alpha = v_g/v_f$ (see Fig. 1).

It is now proposed to fit curves of the type

$$\bar{x}_a = \frac{\gamma \cdot x_m}{1 + x_m(\gamma - 1)} \tag{5}$$

to the new data [5, 6] in which the slip factor γ is a constant at any given pressure. This arbitrary suggestion has the advantage that it provides a simple relation between $\bar{x}_a$ and x_m which extrapolates without discontinuity to the boundary conditions. The values given in

Table 1. Values of α and γ

	Pressure (psia)						
	14·7	250	600	1250	2100	3000	3206
α	1600	99·1	38·3	15·33	6·65	2·48	1·00
γ	246	40·0	20·0	9·80	4·95	2·15	1·00
$\sigma = \frac{\alpha}{\gamma}$	6·45	2·48	1·92	1·57	1·35	1·15	1·00

Table 1 and the curve Fig. 3 were obtained from data from experiments on a 1 in bore horizontal tube. From these the value of γ at any other value of α and corresponding pressure can be found. The experimental curves of Fig. 2 compare with those suggested by Martinelli and Nelson [1].

Independent confirmation of the values of γ thus deduced is provided by Levy [8] and Elrod [9]. Data and curves from these papers are duplicated in Figs. 4 and 5 on which, curves deduced by the methods presented in this paper are superimposed. The correlation achieved is encouraging in view of the simplifying assumptions on which this analysis is based.

Bankoff [10] suggested an equation of the type

$$\frac{1}{x_m} = 1 - \alpha\left(1 - \frac{K}{x_a}\right) \tag{6}$$

which can be re-arranged to give

$$\bar{x}_a = \frac{K\,\alpha\,x_m}{1 + x_m(\alpha - 1)} \tag{7}$$

K, a constant dependent on pressure, was defined by

$$K = 0{\cdot}71 + 0{\cdot}0001\,P.$$

This relation between x_m and $\bar{x}_a$ is somewhat similar to equation (5) but does not satisfy the condition that $\bar{x}_a = 1$ at $x_m = 1$. The application therefore is limited in range. This is apparent from the slope of the two curves of Fig. 4.

The values of x_m covered by the author's work lay mostly in the range $0{\cdot}01 < x_m < 0{\cdot}5$.

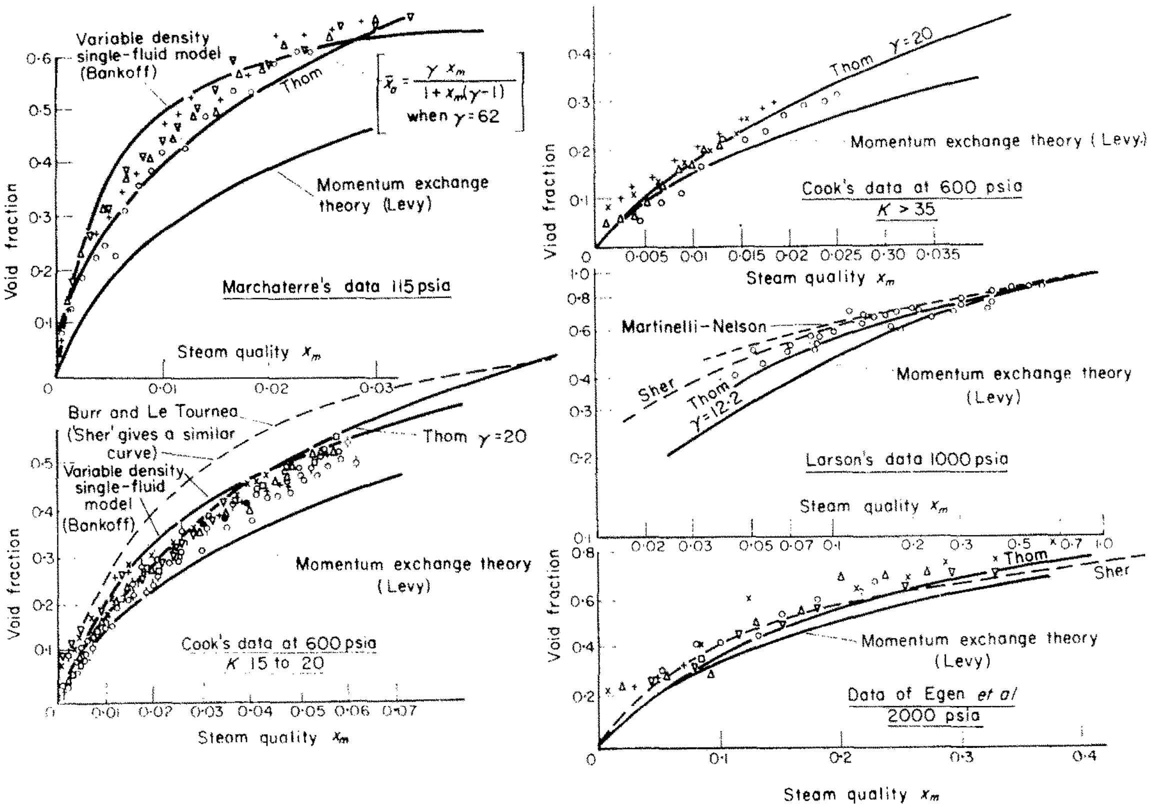

FIG. 4. Comparison between theory and experimental data.

SLIP VELOCITY RATIO

When a two-phase fluid flows through a pipe there is a relative or slip velocity between the phases. This occurs even in a horizontal pipe and at all operating pressures but the effect is greater at lower pressures and smaller at higher pressures. In the pressure range 250–3000 psia, the slip ratio $= V_g/V_f$ between the steam and water phases at the outlet of the pipe decreases from 2·55 at 250 psia to 1·15 at 3000 psia.

At each selected pressure the slip ratio may be taken as almost constant and independent of the quality x_m.

In terms of the quality parameters it may be shown that the slip velocity ratio V_g/V_f and the relative velocity $(V_g - V_f)$ between the phases can be expressed by

$$\frac{V_g}{V_f} = \frac{x_m}{\bar{x}_a} \cdot \frac{1 - \bar{x}_a}{1 - x_m} \left(\frac{v_g}{v_f}\right) \tag{8}$$

and

$$(V_g - V_f) = V_{\text{in}} \left[\frac{x_m}{\bar{x}_a} \cdot \left(\frac{v_g}{v_f}\right) - \frac{1 - x_m}{1 - \bar{x}_a}\right] \tag{9}$$

and substituting for $\bar{x}_a$ in terms of equation (5) gives

$$\sigma = \frac{V_g}{V_f} = \frac{\alpha}{\gamma} \tag{10}$$

and

$$(V_g - V_f) = V_{\text{in}} \left[1 + x_m (\gamma - 1)\right] \left[\frac{\alpha}{\gamma} - 1\right] \tag{11}$$

so that

$$V_f = V_{\text{in}} \left[1 + x_m (\gamma - 1)\right]$$

$$\text{and } V_g = V_{\text{in}} \left[1 + x_m (\gamma - 1)\right] \frac{\alpha}{\gamma}. \tag{12}$$

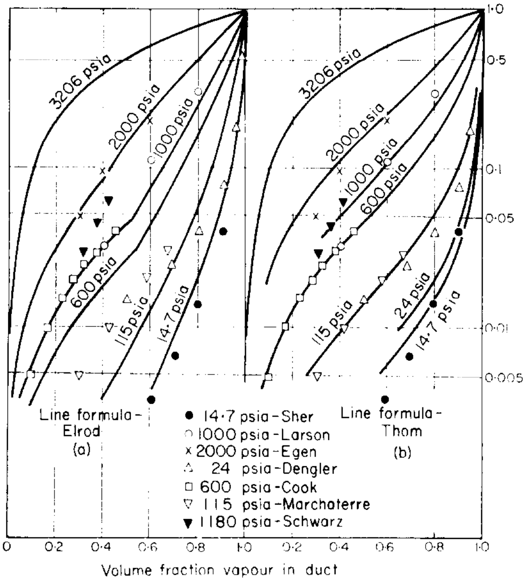

FIG. 5. Comparison between theory and experiment.

PRESSURE DROP DUE TO CHANGE OF MOMENTUM OF THE PHASES

It will be assumed that the specific volume of water at the outlet condition is the same as that at the inlet. Further, it will be assumed that the water at entry is exactly at its saturation temperature according to the pressure at that point of the circuit.

Two extreme cases of outlet condition can exist, the actual condition probably lying between these limits.

(a) Steam and water completely mixed.

(b) Steam and water completely separated.

$$\Delta p_{acl} = \frac{G}{g_c}[(1 - x_m) V_f + x_m V_g - V_{in}] \quad (13)$$

in which V_g and V_f are the velocity of steam and water at the outlet and V_{in} the velocity of water at the inlet.

In the former case, usually referred to as the homogeneous case, $V_f = V_g = V_{out}$ and equation (13) can be expressed

$$_H\Delta p_{acl} = \frac{G}{g_c}(V_{out} - V_{in})$$

or, in terms of the specific volume

$$_H\Delta p_{acl} = \frac{G^2}{g_c}(v_{out} - v_{in}) \quad (14)$$

and, since the mean outlet specific volume is given by

$$v_{out} = v_f + x_m \,.\, v_{fg} \text{ and } v_{in} = v_f$$

this simplifies to

$$_H\Delta p_{acl} = \frac{G^2}{g_c} \,.\, x_m (a - 1) v_f = \frac{G^2}{g_c} r_1 v_f. \quad (15)$$

In the latter case, when slip takes place between the phases so that at the outlet the velocity of the steam V_g is different to that of the water V_f, these quantities have to be eliminated from equation (13). Now

$$V_f = G \,.\, \frac{1 - x_m}{1 - \bar{x}_a} v_f \text{ and } V_g = G \,.\, \frac{x_m}{\bar{x}_a} v_g$$

and hence

$$_S\Delta p_{acl} = \frac{G^2}{g_c}\left[\frac{(1 - x_m)^2}{1 - \bar{x}_a} + \frac{x_m^2}{\bar{x}_a}\left(\frac{v_g}{v_f}\right) - 1\right] v_f = \frac{G^2}{g_c} \,.\, r_2 v_f \quad (16)$$

Comparing equations (16) and (14) gives

$$v_{out} = \frac{(1 - x_m)^2}{1 - \bar{x}_a} v_f + \frac{x_m^2}{\bar{x}_a} v_g. \quad (17)$$

This expression for the specific volume of a two-phase fluid in which slip is occurring has been named the "effective" specific volume and, if $\bar{x}_a$ is related to x_m by equation (5), we get

$$v_{eff} = [1 + x_m (\gamma - 1)]\left[1 + x_m\left(\frac{a}{\gamma} - 1\right)\right] v_f \quad (18)$$

and the "effective" density

$$\bar{\rho}_{eff} = \frac{1}{v_{eff}}$$

This may be compared to the "apparent" density determined directly by gamma-ray measurement.

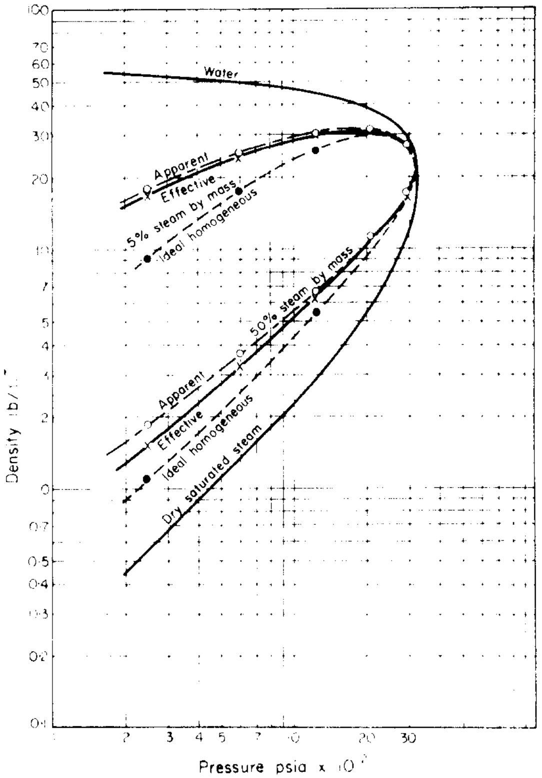

FIG. 6. Comparison between methods of estimating density of a steam and water two-phase mixture.

Thus

$$A\bar{\rho}_a = A_g \rho_g + A_f \rho_f \tag{19}$$

$$\therefore\ \bar{\rho}_a = \rho_f - \bar{x}_a (\rho_f - \rho_g)$$

whence relating $\bar{x}_a$ to x_m by equation (5)

$$\bar{\rho}_a = \frac{\rho_f - x_m (\rho_f - \gamma \rho_g)}{1 + x_m (\gamma - 1)}$$

i.e.

$$\bar{\rho}_a = \frac{1 + x_m [(\gamma/a) - 1]}{1 + x_m (\gamma - 1)} \cdot \frac{1}{v_f} \tag{20}$$

Figure 6 compares $\bar{\rho}_a$ and ρ_{eff} with the ideal homogeneous density defined by

$$\rho = \frac{1}{v} = \frac{1}{v_f + x_m v_{fg}} \tag{21}$$

Returning to equation (16), $\bar{x}_a$ may be eliminated in terms of x_m from equation (5) so that

$$\Delta p_{\text{acl}} = \frac{G^2}{g_c} \left\{ [1 + x_m (\gamma - 1)] \left[1 + x_m \frac{a - \gamma}{\gamma}\right] - 1 \right\} v_f. \tag{22}$$

Note that the subscript H or S is omitted from the pressure drop term of equation (22) since, for homogeneous flow, $\gamma = a = v_g/v_f$ and equation (22) simplifies to equation (15).

For general use equation (22) is too involved but values of the multiplier r_2 as defined by

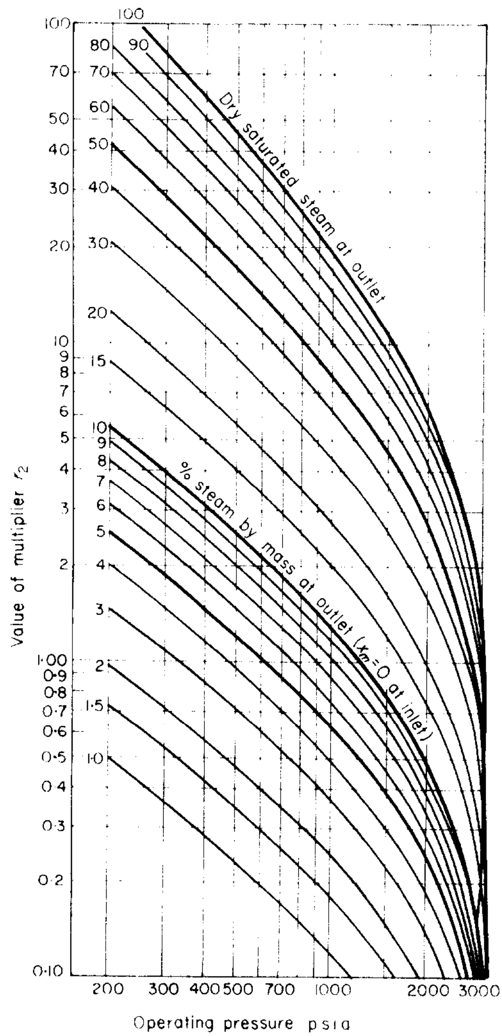

FIG. 7. r_2 acceleration pressure drop multiplier for boiling flow of water and steam.

Table 2. Values of acceleration multiplier r_2

$$\Delta p_{acl} = \frac{G^2}{g_c} \cdot v_f r_2$$

Outlet quality by mass	Pressure (psia)				
	250	600	1250	2100	3000
0·01	0·4125	0·2007	0·0955	0·0431	0·0132
0·015	0·6201	0·3034	0·1441	0·0649	0·0195
0·02	0·8325	0·4055	0·1930	0·0865	0·0261
0·03	1·268	0·6132	0·2903	0·1297	0·0392
0·04	1·710	0·8242	0·3886	0·1739	0·0562
0·05	2·169	1·040	0·4892	0·2182	0·0657
0·06	2·635	1·258	0·5893	0·2582	0·0787
0·07	3·116	1·479	0·6903	0·3076	0·0924
0·08	3·605	1·704	0·7940	0·3521	0·1055
0·09	4·110	1·934	0·8969	0·3975	0·1188
0·10	4·620	2·165	1·001	0·4431	0·1319
0·15	7·365	3·379	1·538	0·6751	0·1996
0·2	10·39	4·678	2·100	0·9139	0·2676
0·3	17·30	7·539	3·292	1·412	0·4067
0·4	25·37	10·75	4·584	1·937	0·5495
0·5	34·58	14·30	5·968	2·490	0·6957
0·6	44·93	18·21	7·448	3·070	0·8455
0·7	56·44	22·46	9·030	3·678	0·9988
0·8	69·09	27·06	10·79	4·312	1·156
0·9	82·90	32·01	12·48	5·067	1·316
1·0	98·10	37·30	14·34	5·664	1·480

$$r_2 = \left\{[1 + x_m(\gamma - 1)]\left[1 + x_m \frac{\alpha - \gamma}{\gamma}\right] - 1\right\} \tag{23}$$

are given in Table 2 and shown in Fig. 7 above which is comparable to Fig. 6 of reference 1.

PRESSURE DROP DUE TO BOILING TWO-PHASE FRICTION

The pressure drop in horizontal tube due to boiling two-phase friction cannot be measured directly. In the W.T.B.A. project [4, 5] it was deduced from the observed total pressure drop on a horizontal tube by deducting the acceleration pressure drop estimated by equation (16). Then having found the two-phase frictional pressure drop, attempts were made to derive the mean value of the two-phase friction factor. Thus

$$\Delta p_{TPF} = 4 f_{TP} \frac{l}{D} \cdot \frac{G^2}{2g_c} \cdot \frac{v_f}{2}\left(\frac{v_{out}}{v_f} + 1\right) \tag{24}$$

or simply

$$\Delta p_{TPF} = f_{TP} \cdot \frac{1}{2f}\left(\frac{v_{out}}{v_f} + 1\right) \Delta p_f. \tag{25}$$

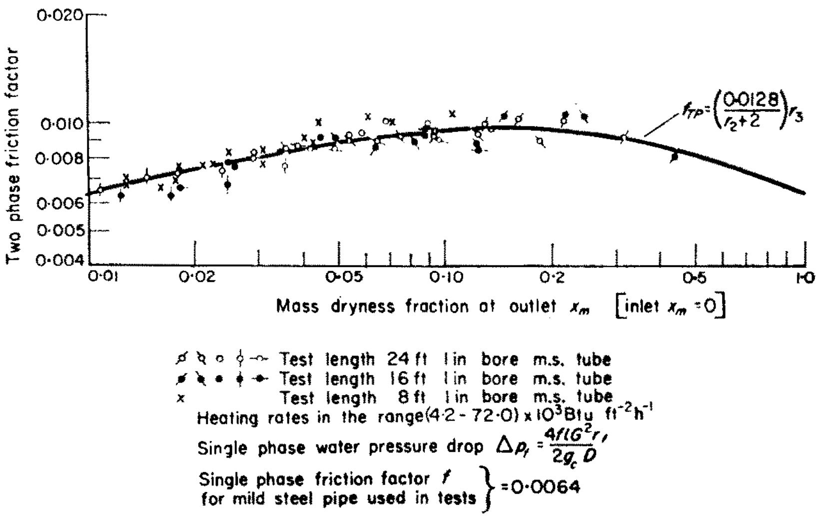

FIG. 8. Two-phase friction factor for boiling flow of water and steam at 600 psia.

Table 3. Values of friction multiplier r_3

$$\Delta p_{TPF} = \Delta p_f \,.\, r_3$$

Outlet quality by mass	Pressure (psia) 250	600	1250	2100	3000
0·00	1·00	1·00	1·00	—	—
0·01	1·49	1·11	1·03	—	—
0·015	1·76	1·25	1·05	—	—
0·02	2·05	1·38	1·08	1·02	—
0·03	2·63	1·62	1·15	1·05	—
0·04	3·19	1·86	1·23	1·07	—
0·05	3·71	2·09	1·31	1·10	—
0·06	4·21	2·30	1·40	1·12	—
0·07	4·72	2·50	1·48	1·14	—
0·08	5·25	2·70	1·56	1·16	1·04
0·09	5·78	2·90	1·64	1·19	1·05
0·10	6·30	3·11	1·71	1·21	1·06
0·15	9·00	4·11	2·10	1·33	1·09
0·2	11·4	5·08	2·47	1·46	1·12
0·3	16·2	7·00	3·20	1·72	1·18
0·4	21·0	8·80	3·89	2·01	1·26
0·5	25·9	10·6	4·55	2·32	1·33
0·6	30·5	12·4	5·25	2·62	1·41
0·7	35·2	14·2	6·00	2·93	1·50
0·8	40·1	16·0	6·75	3·23	1·58
0·9	45·0	17·8	7·50	3·53	1·66
1·0	49·93	19·65	8·165	3·832	1·740

Apart from using the mean outlet homogeneous specific volume which was inadmissible, two other possibilities existed,

(a) the apparent specific volume from void fraction measurements at the outlet.
(b) the effective specific volume defined by equation (18).

Both were tried and it was found that the latter method correlated the various tests which included flow rate, heating rate, heated length and operating pressure as variables. Thus the two-phase friction factor was defined as

$$f_{TP} = \frac{\Delta p_{TPF}}{\Delta p_f}\left\{\frac{2f}{[1 + x_m(\gamma - 1)]\,\{1 + x_m[(a/\gamma) - 1]\} + 1}\right\} \quad (26)$$

or simply

$$f_{TP} = \frac{\Delta p_{TPF}}{\Delta p_f} \cdot \frac{2f}{(r_2 + 2)} \quad (27)$$

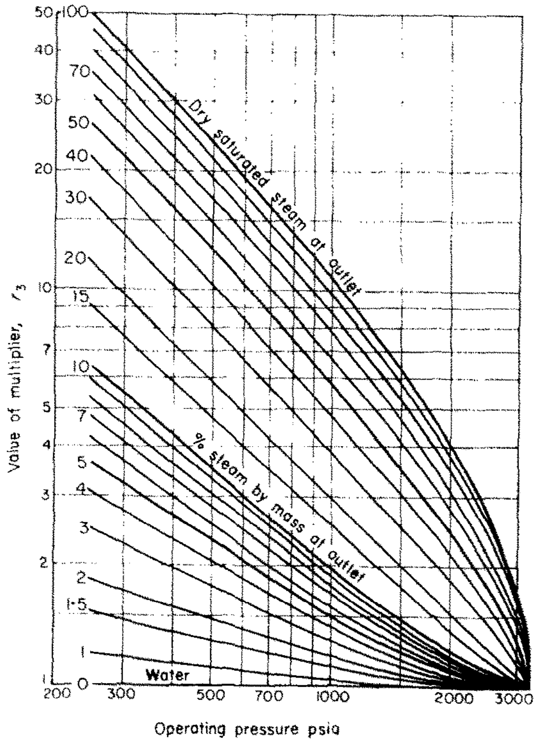

FIG. 9. r_3-friction pressure drop multiplier for boiling flow of water and steam.

Figure 8 shows results at 600 psia and it appears that the two-phase friction factor approximates to a variable function of x_m. Since r_2 is also a function of x_m and pressure, equation (27) was modified so that

$$\Delta p_{TPF} = r_3 \,.\, \Delta p_f \quad (28)$$

Suitable values of r_3 are given in Table 3 and shown in Fig. 9 which are comparable to Table 4 of reference 7, and Fig. 4 of reference 1.

BOILING TWO-PHASE GRAVITATIONAL PRESSURE DROP

The gravitational pressure drop of a boiling two-phase mixture is given by

$$\Delta p_{\text{grav}} = \frac{g}{g_c}\int \frac{dz}{v}. \quad (29)$$

If uniform heating is assumed along the length of the tube and saturated water enters

Table 4. Values of gravitational multiplier r_4

$$\Delta p_{\text{grav}} = \frac{g}{g_c} \cdot \frac{l}{v_f} r_4$$

Outlet quality by mass	Pressure (psia) 250	600	1250	2100	3000
0·00	1·000	1·000	1·000	1·000	1·000
0·01	0·842	0·915	0·956	0·991	0·994
0·015	0·784	0·877	0·937	0·966	0·992
0·02	0·736	0·845	0·918	0·960	0·990
0·03	0·658	0·787	0·882	0·947	0·979
0·04	0·597	0·738	0·850	0·923	0·974
0·05	0·549	0·696	0·823	0·910	0·970
0·06	0·508	0·660	0·795	0·892	0·963
0·07	0·475	0·627	0·771	0·879	0·957
0·08	0·446	0·599	0·746	0·860	0·952
0·09	0·421	0·573	0·724	0·848	0·948
0·10	0·399	0·549	0·705	0·836	0·941
0·15	0·319	0·460	0·624	0·774	0·916
0·2	0·268	0·399	0·558	0·718	0·889
0·3	0·206	0·317	0·468	0·637	0·843
0·4	0·167	0·269	0·406	0·575	0·803
0·5	0·142	0·228	0·357	0·525	0·767
0·6	0·123	0·201	0·321	0·482	0·733
0·7	0·109	0·180	0·291	0·444	0·702
0·8	0·098	0·163	0·267	0·413	0·675
0·9	0·089	0·149	0·248	0·389	0·651
1·00	0·081	0·136	0·229	0·364	0·626

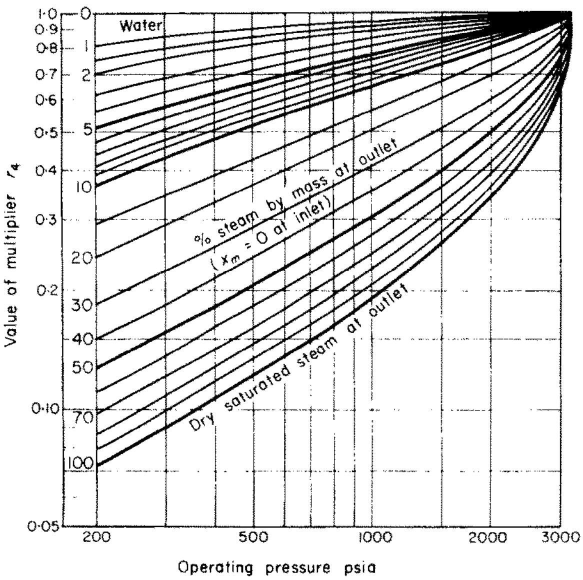

FIG. 10. r_4 gravitational pressure drop multiplier for boiling flow of water and steam.

at the inlet then for ideal homogeneous flow, equation (29) can be integrated to give

$$_H\Delta p_{\text{grav}} = \frac{g}{g_c} \cdot \frac{l}{v_f} \cdot \frac{1}{R-1} \log_e R. \tag{30}$$

The problem of two-phase flow is complex. The mean density or mass dryness fraction of the mixture leaving the outlet of the boiling tube may be evaluated by a heat balance. Alternatively the apparent density of the two-phase mixture may be measured by a gamma-device.

Then from equation (20)

$$\frac{1}{v_a} = \frac{1 + x_m[(\gamma/a) - 1]}{1 + x_m(\gamma - 1)} \cdot \frac{1}{v_f} \tag{31}$$

and since equation (29) may be written

$$\Delta p_{\text{grav}} = \frac{g}{g_c} \cdot \frac{l}{(x_m)_{\text{out}}} \int \frac{\mathrm{d}x_m}{v}$$

we get, after substitution and integration

$$\Delta p_{\text{grav}} = \frac{g}{g_c} \cdot \frac{l}{v_f} \left\{ \frac{(\gamma/a - 1)}{\gamma - 1} + \frac{\gamma - (\gamma/a)}{(\gamma - 1)^2} \cdot \frac{\log_e [1 + x_m(\gamma - 1)]}{x_m} \right\} \tag{32}$$

or simply

$$\Delta p_{\text{grav}} = \frac{g}{g_c} \cdot \frac{l}{v_f} r_4. \tag{33}$$

Values of r_4 are given in Table 4 and shown in Fig. 10 as a function of pressure and outlet steam quality.

SUMMARY

In the foregoing theory water entering the tube was assumed to be at saturation temperature. Thus evaporation, with net generation of steam, started at once.

The pressure drop in self-consistent units is given by

$$\Delta p = \Delta p_{\text{acl}} + \Delta p_{\text{frict}} + \Delta p_{\text{grav}}$$

$$= \frac{1}{g_c} v_f G^2 r_2 + \frac{4fl}{2g_c D} v_f G^2 r_3 + \frac{g}{g_c} \frac{l}{v_f} r_4. \tag{34}$$

Figure 11 compares values calculated from equation (34) with the later data of Haywood *et al.* [5] for a vertical tube.

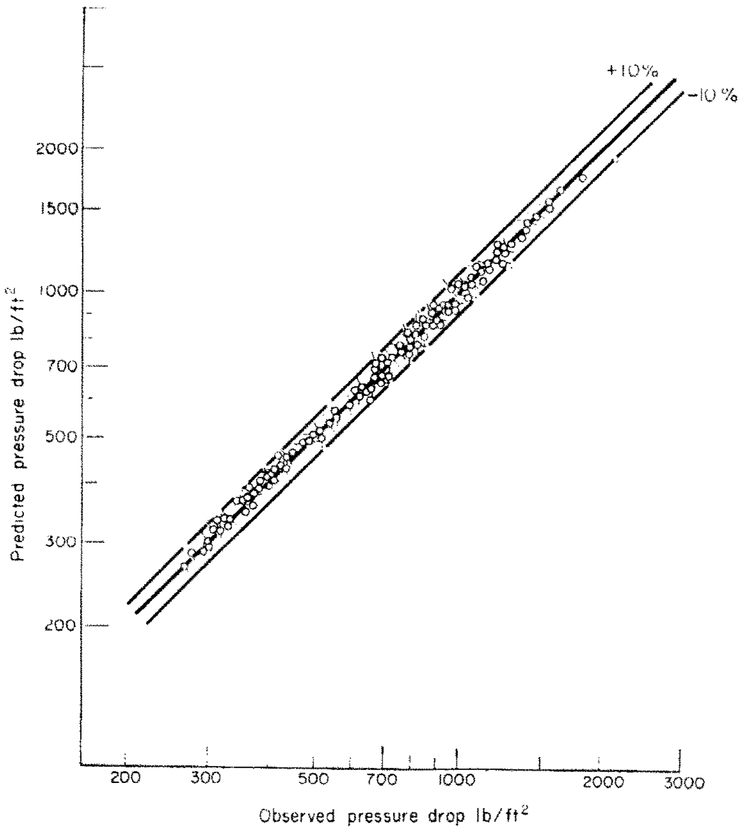

FIG. 11. Forced circulation boiling of water; 250–2100 psia 1 in bore vertical pipe.

EXAMPLE

Consider a practical case in which subcooled water enters a uniformly heated vertical tube of diameter D' in at a mass flow rate of G' lb/ft^2 h. Conditions are assumed to be:

Boiler pressure, 1000 psia;
Tube bore (D'), 0·5 in;
Tube length (l), 5 ft 1 in;
Heat flux through inner surface of tube, 200 000 Btu/ft^2 h;
Water flow, 700 lb/h;
Mass flow (G'), 515 000 lb/ft^2 h;
Total heat added to fluid (Q), 190 Btu/lb;
Water temperature at inlet (T_i), 500°F.

A length of tube is required to preheat the water to saturation temperature and, for a total heat addition of Q Btu/lb of fluid in the total length l of the tube, the liquid heating length l_i is given by

$$l_i = \frac{(h_f - h_i)\, l}{Q} \tag{35}$$

where h_f is the enthalpy of the water at saturation temperature and h_i the enthalpy at inlet temperature.

In the rest of the tube steam is generated by saturated boiling and the mass dryness fraction x_m at the outlet can be calculated from a heat balance as

$$x_m = \frac{h_i - h_f + Q}{h_{fg}} \tag{36}$$

where h_{fg} is the increase in enthalpy with evaporation.

Using the values given above the quantities l_i and x_m evaluate as follows

$$l_i = \frac{(542{\cdot}4 - 524{\cdot}1)\,5{\cdot}08}{190} = 0{\cdot}49 \text{ ft}$$

and

$$x_m = \frac{524{\cdot}1 - 542{\cdot}4 + 190}{649{\cdot}4} = 0{\cdot}265.$$

The total pressure drop along the channel is the sum of six terms which can be calculated separately in practical units of psi from the formulae as follows:

(a) *Preheating section*

$$\Delta p'_{\text{acl}} = 1{\cdot}664 \times 10^{-11}\,(v_f - v_i)\,(G')^2 \qquad (37)$$

$$\Delta p'_{\text{frict}} = 1{\cdot}997 \times 10^{-10}\,(v_f + v_i)\frac{f\,l_i}{D'}\,(G')^2 \qquad (38)$$

$$\Delta p'_{\text{grav}} = 1{\cdot}389 \times 10^{-2}\,\frac{l_i}{v_f + v_i}. \qquad (39)$$

(b) *Boiling section*

$$\Delta p'_{\text{acl}} = 1{\cdot}664 \times 10^{-11}\,v_f\,(G')^2\,r_2 \qquad (40)$$

$$\Delta p'_{TPF} = 3{\cdot}994 \times 10^{-10}\,v_f\,\frac{f}{D'}\,(l - l_i)\,(G')^2\,r_3 \qquad (41)$$

$$\Delta p'_{\text{grav}} = 6{\cdot}944 \times 10^{-3}\,\frac{l - l_i}{v_f}\,r_4. \qquad (42)$$

In equations (38) and (41) f is the Fanning friction factor for single phase water flow in the tube at a mass velocity G' lb/ft^2 h and at the appropriate pressure and temperature conditions. The multipliers r_2, r_3 and r_4 are interpolated for the appropriate value of x_m from Figs. 7, 9 and 10 respectively. Frequently, however, when a range of flow conditions has to be evaluated at the same operating pressure it will be convenient to plot the respective multipliers as shown in Fig. 12. v_f and v_i are the specific volumes of water at saturation and inlet conditions respectively.

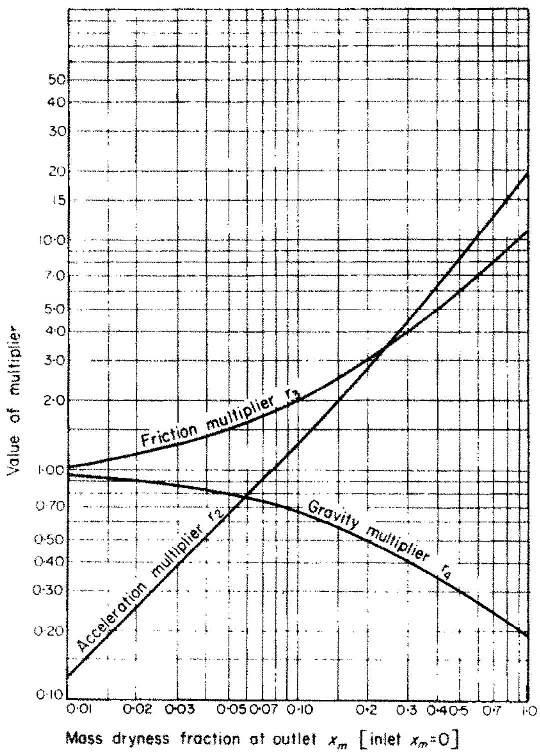

FIG. 12. Acceleration, friction and gravity pressure drop multipliers for boiling flow of water and steam at 1000 psia.

Again using the values given equations (37) to (42) are then evaluated as follows:

(a) *Preheating section*

$$\Delta p'_{\text{acl}} = 1{\cdot}664 \times 10^{-11}\,(0{\cdot}0216 - 0{\cdot}0204)\,(5{\cdot}15 \times 10^5)^2 = 0{\cdot}005 \text{ psi}$$

$$\Delta p'_{\text{frict}} = 1{\cdot}997 \times 10^{-10}\,(0{\cdot}0216 + 0{\cdot}0204)\,\frac{0{\cdot}0059 \times 0{\cdot}49}{0{\cdot}5}\,(5{\cdot}15 \times 10^5)^2 = 0{\cdot}129 \text{ psi}$$

$$\Delta p'_{\text{grav}} = 1{\cdot}389 \times 10^{-2}\,\frac{0{\cdot}49}{0{\cdot}0216 + 0{\cdot}0204} = 0{\cdot}162 \text{ psi}$$

$$= 0{\cdot}296 \text{ psi}$$

(b) *Boiling section*

$$\Delta p'_{acl} = 1{\cdot}664 \times 10^{-11} \times 0{\cdot}0216\,(5{\cdot}15 \times 10^5) \times 3{\cdot}85 = 0{\cdot}366 \text{ psi}$$

$$\Delta p'_{TPF} = 3{\cdot}994 \times 10^{-10}\,\frac{0{\cdot}0216 \times 0{\cdot}0059}{0{\cdot}5}\,(5{\cdot}08 - 0{\cdot}49)\,(5{\cdot}15 \times 10^5)^2 \times 3{\cdot}64 = 0{\cdot}452 \text{ psi}$$

$$\Delta p'_{grav} = 6{\cdot}944 \times 10^{-3}\,\frac{(5{\cdot}08 - 0{\cdot}49)}{0{\cdot}0216} \times 0{\cdot}44 = 0{\cdot}650 \text{ psi}$$

$$1{\cdot}468 \text{ psi}$$

Estimated total pressure drop $= 1{\cdot}77$ psi

ACKNOWLEDGEMENTS

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APPENDIX

In an evaporating circuit there will usually be an unheated section at the outlet end along which two-phase flow will take place. Over a short length of tube, the static pressure can be regarded as approximately constant and only the frictional term in the case of a horizontal tube, or frictional and gravitational terms in the more general case of an inclined tube, need be considered.

As in the case of boiling flow, the two-phase frictional pressure drop is related to the corresponding water only pressure drop for the same mass flow rate. Thus:

$$\frac{\Delta p_{TPF}}{\Delta p_f} = r_5$$

Numerical values of the dimensionless term r_5 are given in Table 5 and shown in Fig. 13.

Figures 14 and 15, copied from a paper by Muscettola [13], show how this prediction compares with data from C.I.S.E. (Milan, Italy).

Table 5. Values of friction multiplier r_5
Flow of water and steam in unheated tubes.

$$\Delta p_{TPF} = \Delta p_f \, . \, r_5$$

Outlet quality by mass	Pressure (psia) 250	600	1250	2100	3000
0·00	1·00	1·00	1·00	—	—
0·01	2·12	1·46	1·10	—	—
0·015	2·71	1·60	1·16	—	—
0·02	3·22	1·79	1·22	1·06	—
0·03	4·29	2·13	1·35	1·11	—
0·04	5·29	2·49	1·48	1·16	—
0·05	6·29	2·86	1·62	1·21	1·92
0·06	7·25	3·23	1·77	1·26	1·03
0·07	8·20	3·61	1·92	1·31	1·04
0·08	9·15	3·99	2·07	1·37	1·05
0·09	10·1	4·38	2·22	1·42	1·06
0·10	11·1	4·78	2·39	1·48	1·08
0·15	15·8	6·60	3·03	1·75	1·16
0·2	20·6	8·42	3·77	2·02	1·24
0·3	30·2	12·1	5·17	2·57	1·40
0·4	39·8	15·8	6·59	3·12	1·57
0·5	49·4	19·5	8·03	3·69	1·73
0·6	59·1	23·2	9·49	4·27	1·88
0·7	68·8	26·9	10·19	4·86	2·03
0·8	78·7	30·7	12·4	5·45	2·18
0·9	88·6	34·5	13·8	6·05	2·33
1·0	98·86	38·30	15·33	6·664	2·480

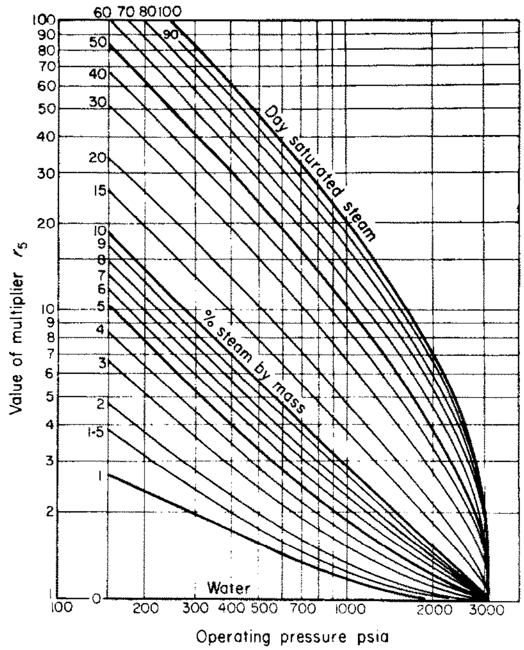

FIG. 13. Friction pressure drop multiplier for flow of water and steam in unheated tubes.

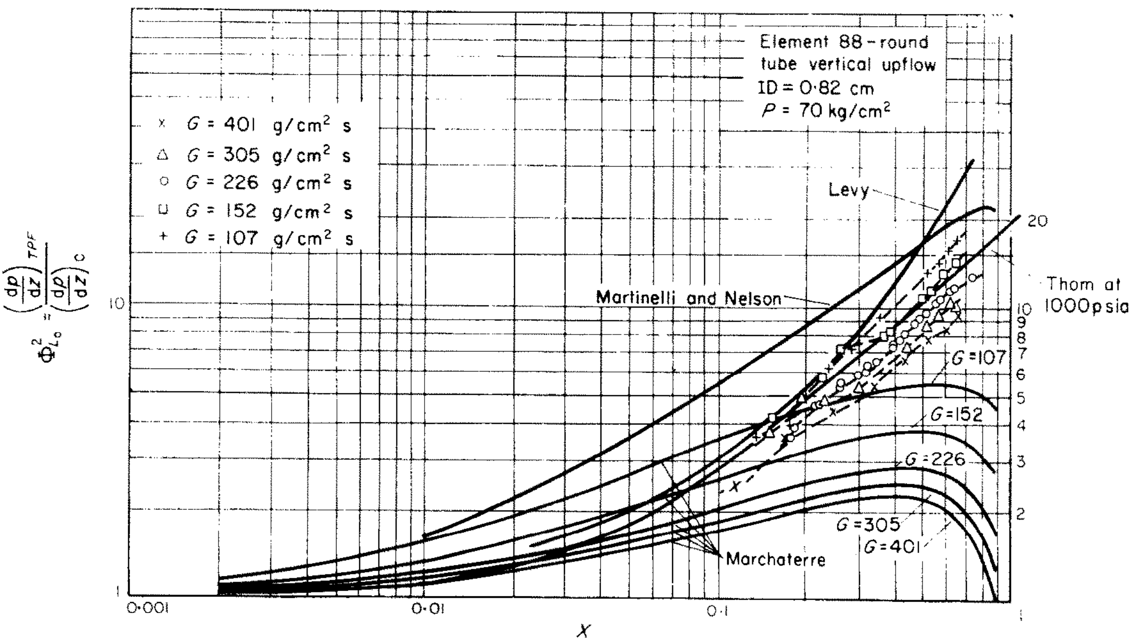

FIG. 14.

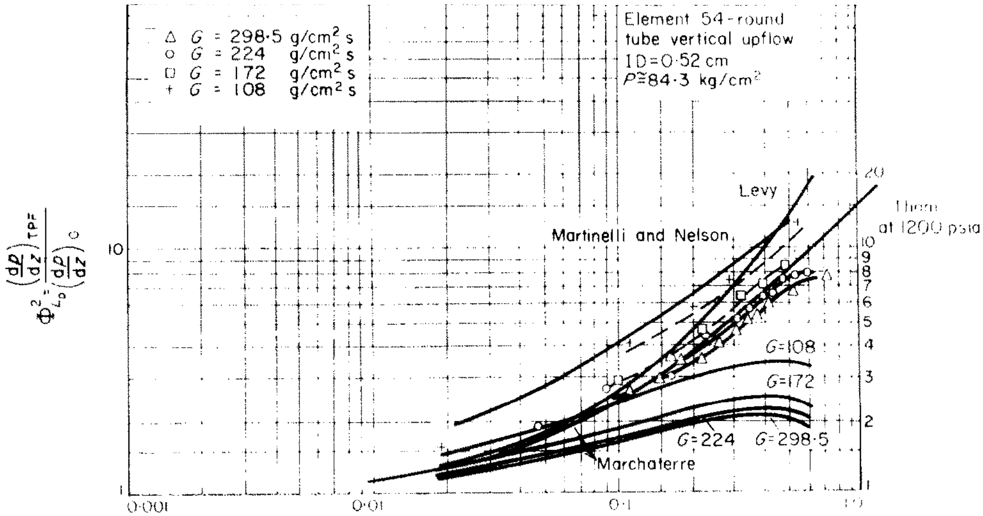

FIG. 15.

The gravitational term can be calculated from

$$\Delta p_{\text{grav}} = \frac{g}{g_c} \cdot \frac{l \sin \theta}{v_a}$$

in which the apparent specific volume v_a of the two-phase mixture is given by

$$v_a = v_f \left\{ \frac{1 + x_m (\gamma - 1)}{1 + x_m [(\gamma/\alpha) - 1]} \right\}$$

Values of α, the specific volume ratio, are calculated from steam tables, and γ is found from Fig. 3.

Résumé—Un schéma simplifié est donné pour le calcul de la chute de pression pendant la circulation d'un mélange diphasique d'eau en ébullition et de vapeur. La méthode suit celle proposée par Martinelli et Nelson qui a été étendue pour inclure le terme de gravité dans des tubes d'évaporation verticaux. On donne des courbes à partir desquelles les pertes par frottement, accélération et gravité peuvent être estimées pourvu que la qualité de sortie ait été calculée à partir d'un bilan de chaleur. Ces courbes sont basées directement sur le résultat expérimental de la recherche sur la circulation dans les chaudières financée à l'Université de Cambridge par l'Association des Fabricants de chaudières à tubes d'eau.

Une annexe donne les renseignements pour calculer la chute de pression pendant l'écoulement diphasique d'eau et de vapeur le long d'un tube non chauffé.

Zusammenfassung—Zur Berechnung des Druckabfalls in Rohren während des Umlaufs eine Zweiphasengemisches aus Wasser und Wasserdampf wird ein vereinfachtes Schema angegeben. Die Methode folgt von Martinelli und Nelson mit einer Erweiterung, die den Gravitationsausdruck für senkrechte Dampfrohre berücksichtigt. Kurven sind angegeben, aus denen Reibungs-, Beschleunigungs- und Gravitationsverluste bei bekannten, aus einer Wärmebilanz errechnetem Austrittszustand entnommen werden können. Diese Kurven beruhen unmittelbar auf Ergebnissen einer experimentellen Untersuchung, die an der Universität von Cambridge mit Unterstützung der Water-Tube Boilermakers' Association durchgeführt wurde.

Im Anhang werden Angaben für die Berechnung des Druckabfalles während einer Zweiphasensträuung aus Wasser und Wasserdampf im unbeheizten Rohr gemacht.

Аннотация—Предлагается упрощенная схема расчета перепада давления при циркуляции двухфазной смеси кипящей воды и водяного пара. Метод аналогичен предложенному Мартинелли и Нельсоном, но с учетом гравитационного члена в вертикальных испарительных трубах. Приводятся кривые для вычисления потерь на трение, ускорение и гравитацию при условии, что величины на выходе вычислялись из теплового баланса. Эти кривые получены непосредственно из экспериментальных данных по исследованию циркуляции в паровых котлах, проведенного в Кембриджском Университете по заданию Ассоциации предприятий, выпускающих водотрубные котлы.

В приложении приводятся данные для вычисления перепада давления в двухфазном течении вода-пар вдоль ненагретой трубы.